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Analysis of the influence of loading and the plasticity index on variations in surface roughness between two flat surfaces

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ABSTRACT

Contact between two highly loaded flat surfaces is examined. In these experiments, a polished, hard metal surface was pressed against a rough aluminium alloy surface. The goal of the experiments was to flatten the peaks on the rough surface by plastic deformation. The experiments elucidated the relation between the normal load and the plasticity index, ψ, and the plastic deformation of the surface roughness peaks in the softer material. The results showed a significant reduction in surface roughness. Increasing the load caused a gradual reduction in the surface peaks. However, based on the values of the plasticity index ψ, it can be concluded that the peaks of the softer material underwent plastic deformation, regardless of the load.

1. Introduction

Finishing is a highly important segment of any manufacturing process. To increase the reliability and the service life of components, modern industry employs various technologies for modifying the surface of a workpiece such as surface treatments, thermochemical and chemical treatments, ion implantation and plastic forming. Cold plastic forming of material is a process in which the material yields under high compressive loads that exceed the elastic limit of the material. Plastic forming is also considered an efficient way to reduce the friction coefficient between tribological contact pairs. Plastic forming reduces the maximum heights of the surface roughness peaks [1,2].

Surface roughness is important for the proper functioning of mechanical assemblies, resistance to wear, the service life of assemblies and fatigue strength [3]. Measurements of the microscopic surface geometry can be used to describe the contact interface between surfaces and the contact interface surface roughness (form, distance and amplitude) [4]. Among the most common surface roughness parameters are R_a, R_q, R_{sk} and R_{ku}. The arithmetic average of the roughness profile, R_a, reflects the general differences between roughness heights. However, this parameter provides no information on the wavelength or the small differences in the peaks. The root mean square roughness, R_q, is more informative than R_a regarding the deviations of the peak heights from the mean profile line. The surface skewness, R_{sk}, defines the distribution of the valleys and the peaks, and kurtosis, R_{ku}, describes the sharpness of the probability density of the profile [5]. Surface roughness is also important for load transmission. Surfaces with very high R_q values are able to sustain small loads, whereas surfaces with medium or small R_q values can tolerate high loads without surface damage [6,7]. When a normal load is applied to contact surfaces, the contact occurs only between the peaks on the surfaces [8]. Depending on the magnitude of the normal force, the peaks may undergo elastic, elasto-plastic or plastic deformations. At the initial contact, the peaks elastically deform. As the load increases, the peaks are subject to plastic deformation. Consequently, when rough surfaces are in contact, some percentage of the peaks deform elastically and the remainder deform plastically [9,10]. The deformations in the contact area are opposite in direction to the applied load. If the deformation is proportional to the surface topography, then the effect of the normal load is strictly dependent on the magnitude of the roughness peak heights [11,12].

A review of the literature has established that finishing by indentation – which causes plastic deformation in the surface layer – is the most frequently used method besides conventional machining to reduce surface roughness [13–28]. Analytical and numerical modelling of elastic deformation and contact surfaces has been the subject of numerous investigations. Tadic et al. [15] analysed the influence of indentation performed with various tool geometries on the fixture load capacity. Many researchers have opted for spherical tool tips [16,21–23,27,28], and some have used

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Nomenclature

A	surface area
a	side of intender
E^*	Young's modulus of the softer material
F_N	normal load
F_w	force of weights
H	hardness of the softer material
h	height of specimen
k	peak curvature
L	evaluation length
l	sampling interval
p	nominal contact pressure
R_a	arithmetic average height
R_c	correlation coefficient
R_p	maximum height of peaks
R_q	root mean square roughness
R_v	maximum depth of valleys
r	mean radius of asperities
w	width of specimen
ν	Poisson's ratio
σ	standard deviation of the asperity heights
ψ	plasticity index

conical ones [18]. However, only a few authors have addressed the issue of using a flat surface for the tool tip geometry [19,20]. Greenwood and Williamson [19] studied contact between two flat, elastic surfaces, one rough and the other smooth. They assumed that the rough surface was covered in spherical bumps and that the distribution of the peak heights was normal (i.e., Gaussian). Li et al. [21] performed a contact analysis based on Hertz theory, where the roughness peaks undergo elastic deformation. This model of elastic peak deformation has been used in other studies to model contact between two rough, spherical surfaces [23], and the contact between two nominally rough, flat surfaces [24].

Investigations of plastic deformation on a flat surface have also been described in the literature [25–30]. It was shown that the transitions from elastic deformation into the elasto-plastic deformation and from elasto-plastic deformation into completely plastic deformation depend not only on the stress condition but also on the plasticity index, ψ, which is specific to the material properties and the topography of the contact surfaces [25]. Presented in the paper were critical values of ψ which lead to pronounced plastic deformations of surface roughness peaks. It is known that the values of the surface roughness parameters and consequently the calculation of the plasticity index depend on the sampling interval used [31–36]. Greenwood [31] showed that an increase in the sampling interval from 1 μm to 15 μm decreased the mean peak curvature of a ground steel surface from 300 mm^{-1} to 5 mm^{-1} and concluded that the value of the sampling interval should be reported together with the calculated roughness parameters. Pawlus and Zelasko [32] analysed the relationship between the sampling interval and the contact mechanics of rough, isotropic surfaces to determine the appropriate sampling interval. They analysed three computer-generated surfaces and one real surface with four different sampling intervals. Larger sampling intervals reduced the separation between the contact surfaces under a given load and the plasticity index. Scaraggi et al. [33] showed that the number of peaks in the roughness profile is strongly affected by the fractal dimensions of the surface. Zavarise et al. [34] addressed the problem of the dependence of the mechanical responses given by various micromechanical models of contact between rough surfaces on resolution; i.e., stochastic models of contact are dependent on the resolution at which the surfaces are described. Kucharski and Starzynski [35] studied the influence of the sampling interval on the following parameters: the bearing (Abbott) curve and the amplitude, the radii, the density and the distribution of the asperities. When the sampling interval was increased, the mean asperity radius increased from 2 μm to several hundred micrometres, which indicated an acute sensitivity of this parameter to the size of the sampling interval. This result also shows the importance of the proper interpretation of roughness measurements. Recommendations for choosing the sampling interval size have been provided by a number of authors [29,32,36]. For example, Poon and Bhushan [36] suggest using the value $0.4 \cdot \beta^*$ as the sampling interval, where β^* is the correlation length. Subsequently, Greenwood [37] suggested the use of quantities that are independent of the sampling interval such as the autocorrelation function (ACF) or the spectral density.

The Greenwood–Williamson theory of contact [25] has been widely accepted, but Greenwood and Wu [37] questioned the validity of the 3-point peak presented in their previous paper. They favoured Archard's [38] concept in which roughness is represented as 'protuberances on protuberances on protuberances', which is suggestive of a fractal but was developed some 20 years before term fractal surface was invented. Greenwood and Williamson [39] demonstrated that micro-asperities affect contact only in the very early stages, after which the nature of the contact will depend on the large-scale geometry of the asperities.

Moore [40] pressed a smooth steel cylinder against a face-turned copper plate and found that the peaks in the copper surface remained clearly visible in the indentation profile, which demonstrated the remarkable persistence of the surface asperities. Asperity persistence has been studied by a number of researchers [41–43], and it was concluded that asperity persistence does not depend on the particular metal in contact. The real area of contact, A_r, is smaller than the nominal area, A_n, and even under very high loads, the peaks on the rough surfaces are not flattened. Pullen and Williamson [41] and Williamson and Hunt [42] suggested that the ratio A_r/A_n for homogeneous solids is one half (50%). Poon and Sayles [44] analysed the influence of surface roughness parameters (the RMS value of the height distribution and the correlation length) to show that three types of surface contact can occur, i.e., predominantly plastic asperity contact, predominantly elastic asperity contact and nearly total surface contact under an elastic state of stress. Johnson [29] suggested that there is a scale of roughness below which the asperities are immediately destroyed by plastic deformation when two surfaces are brought into contact and, in general, do not contribute significantly to the contact pressure.

In this study, contact between two flat surfaces, one smooth and very hard and the other rough and relatively soft, is considered. The rationale behind this study is that under increasing loads, the rough surface should become smoother; i.e., the surface roughness of the rough surface should approach that of the smooth surface. It is expected that the process of indentation will cause plastic deformation of the peaks on the rough contact surface. The plastically deformed peaks on the softer material will flow and fill in the valleys of the surface profile, resulting in a reduction of the surface roughness. The experimental results confirmed this prediction.

2. Theoretical model

Contact between two flat surfaces is illustrated in Fig. 1, where the upper surface, the indenter, is assumed to be perfectly smooth

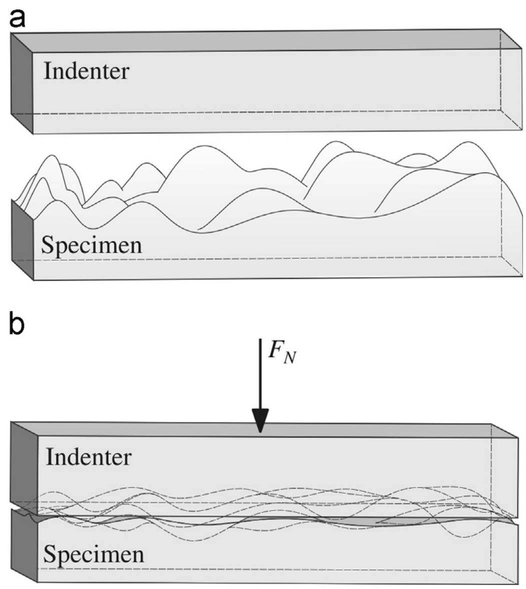

Fig. 1. Contact surfaces: (a) no load and (b) loaded.

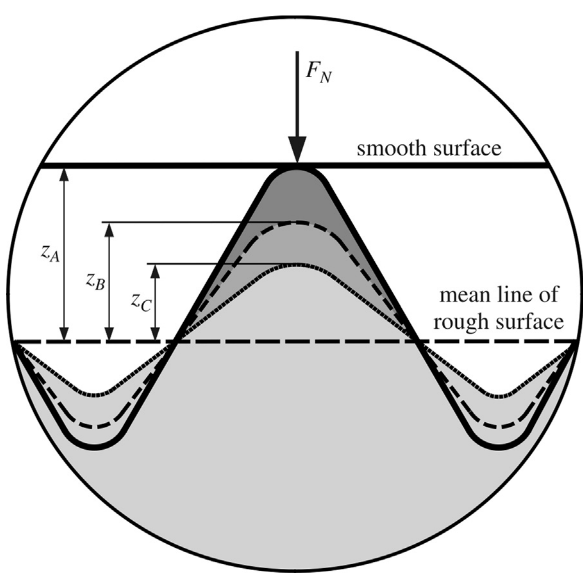

Fig. 2. Detailed view of the plastic flow of surface roughness peaks.

and the lower surface, i.e., the specimen, has a rough surface. The flat indenter surface is pressed into the rough specimen surface using various loads. Shown in Fig. 1a is the initial stage of indentation, where there is no load on the specimen. Fig. 1b illustrates contact under a load.

It is assumed that under a load, a plastic flow of material from the peaks to the valleys in the surface occurs. A detailed rendering of the surface roughness profile prior to the introduction of a load (z_A) and upon loading (z_B, z_C) is shown in Fig. 2.

It is supposed that because of plastic flow in the specimen, material from the peaks will fill in the valleys, thus reducing the overall surface roughness. As shown in Fig. 1, the topography of the specimen contact surface features peaks of various heights. The theoretical model proposed by Greenwood and Williamson [25] assumes that the peaks are of a spherical shape with radius r and the distribution of their heights is Gaussian, i.e., with the probability density function $\phi(z)$:

$$\phi(z)=\frac{1}{\sqrt{2\pi}}e^{-z^2/2\sigma^2} \tag{1}$$

where σ is the standard deviation and z is the height of surface roughness peaks measured from the mean profile line.

The mean profile radius, r, defines the curvature of the peaks. This parameter is based on the individual radii of curvature r_i within a profile. A point in the profile can be considered a peak when its height is greater than that of the two closest points, and the peak curvature k_i, i.e., the reciprocal of the radius r_i, at height z_1 is calculated as [5,31,35]

$$k_i=\frac{1}{r_i}=\frac{2z_i-z_{i-1}-z_{i+1}}{l^2} \tag{2}$$

where z_i is the height of the peak, z_{i-1} and z_{i+1} are the heights of the two closest points, and l is the sampling interval length.

The mean peak radius, r, is calculated according to following relation:

$$r=\frac{1}{n}\sum_{i=1}^{n} r_i \tag{3}$$

where n is the number of peaks.

Depending on the roughness magnitude, pitch and form, contact is established over a finite number of roughness peaks. The roughness constrains the contact between two bodies to very small contact surfaces. Accordingly, the real contact surface under a normal load depends on the number of micro-scale contact surfaces. As the load increases, the number of contact surfaces increases. The actual contact surface area is the result of the deformation of the surface roughness peaks that are in contact. These deformations can be elastic, elasto-plastic or fully plastic, depending on the magnitude of the load, the material properties, the contact surface topography and the plasticity index.

According to the theory of Greenwood and Williamson [25], the plasticity index defines the degree of plastic deformation within the contact surface area:

$$\psi=\frac{E^*}{H}\sqrt{\frac{\sigma}{r}} \tag{4}$$

where H is the hardness of the softer material, σ is the standard deviation of the peak heights, r is the mean radius of the profile peaks, and the Hertz elastic modulus E^* can be calculated as [25,32,34]

$$E^*=\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)^{-1} \tag{5}$$

where E_i and ν_i ($i=1, 2$) are Young's moduli and the Poisson ratios for the two contact materials, respectively. If one of the contacting surfaces is much more elastic than the other, E^* equals the plane-stress modulus for that material [25]:

$$E^*=\frac{E}{(1-\nu^2)} \tag{6}$$

A high E^*/H ratio indicates a high plasticity index. In that case, for the deformations to remain within the elastic range, it is necessary to lower either the surface roughness or the contact pressure. Accordingly, only finely polished surfaces undergo elastic deformation; others will deform plastically.

The authors of this paper maintain that there is a theoretical foundation to support the claim that surface roughness can be significantly reduced by simply pressing a smooth, hard indenter into a rough specimen of lower hardness.

3. Experimental investigation

In these experiments on the use of plastic forming to reduce surface roughness, the results were based on measurements of the surface roughness peaks on the specimen. The indenter, which had a smooth, flat surface, was subjected to a normal force to treat the rough, flat surface of the sample. The indenter was insert for machine cutting tools, WH05ZT, and the specimen was made of the aluminium alloy EN AW-6082 T651 (ISO AlSi1M), for which the material properties are presented in Table 1. The contact interface was loaded with a static force. In the specially designed fixture (Fig. 3), which essentially consists of a lever mechanism, the desired loads are generated by changing the nominal weights. The instrument used to take measurements in these experiments is described in more detail in [15].

Lee-Prudhoe et al. [43] investigated asperity persistence under heavy loads for a smooth, hard surface indenting a rough aluminium surface using a similar test rig. Unlike the experiments of Lee-Prudhoe et al. [43], in which the indenter was a barrel-shaped roller that was rolled over rectangular, flat specimens, two flat surfaces were used in this study.

The experiments were conducted as follows. Weights with nominal masses of 10–120 kg in 5 kg increments were used to produce the contact loads. To calculate the indenting load, the masses of the lever and the axle that carried the weights were also included. The nominal indenting forces, F_N, were between 600 N and 15.3 kN. The normal load, F_N, divided by the indenter surface area resulted in pressures ranging from 40.1 MPa to 944.8 MPa. The force, F_N, was applied on the specimen by the fixture via the indenter. Accordingly, the geometry of the specimen was determined by the geometry of the indenter. For each normal load, indenting was performed in various spot areas on the specimen, which resulted in a total of 24 tests. The initial surface roughness of the indenter and the specimen, as well as surface roughness of the specimen after indentation, were measured. The dimensions of the indenter and the specimen and the measurements of the initial values of the surface roughness are given in Table 2.

The changes that occurred on the treated surfaces as a result of indenting are discussed in the following section.

Table 1
Chemical, mechanical and physical properties of the specimen material EN AW-6082 T651 (ISO AlSi1M).

Chemical compositions								
Elements	Si	Fe	Mn	Mg	Cr	Zn	Ti	Rest
wt%	0.9	0.5	0.6	0.9	0.25	0.2	0.1	Al
Mechanical properties				Index		Unit		Amount
Tensile strength				R_m		MPa		275–300
Yield strength				$R_{p0.2}$		MPa		240–255
Brinell hardness				HB				95
Knoop hardness				HK				120
Physical properties				Index		Unit		Amount
Specific gravity				G		g/cm^3		2.7
Elastic modul				E		GPa		69
Poisson's ratio				ν				0.33
Lin. Coeff. of therm. exp. (20–100 °C)				α		μm/m °C		23.4
Thermal conductivity				κ		W/mK		165–185

4. Results and discussion

The experimental results comprise the data on the nominal pressure used to perform the surface treatment, the data on the resulting surface roughness parameters, and the influence of the plasticity index ψ on the plastic deformations of the surface roughness peaks. Diagrams are used to illustrate the effect of the nominal pressure on the surface roughness height parameters and the plasticity index. To assess the surface topography, the following parameters were used: R_a, R_q, R_p, R_v and r. Surface roughness profiles were measured using a Rank Taylor Hobson (UK) Talysurf 6 profiler with a variable-inductance transducer and a 2 μm stylus tip. The selected evaluation length L=1.25 mm contained 1150 sampling points, which resulted in a sampling interval l=1.087 μm. Using Eq. (1), a statistical analysis of the heights of the surface roughness peaks showed that the data have a Gaussian distribution. The Gaussian distribution assumption was tested using the Kolmogorov–Smirnov test [45,46] for the indenter and specimen surface roughness data (Fig. 4 and Fig. 5). To verify the hypothesis on the equality of the empirical and theoretical distributions (in this case, the Gaussian distribution), two hypotheses are made: H_0 is the null hypothesis, which states that there is no significant difference, and H_a is the alternative hypothesis, which states that there are significant differences. A statistical analysis was performed using the software StatSoft Statistica 12,

Table 2
Surface and geometry properties of indenter and specimen.

Contact body	R_a (μm)	R_q (μm)	R_p (μm)	R_v (μm)	l (mm)	w (mm)	h (mm)	a (mm)	A (mm^2)
Intender	0.13	0.17	0.35	0.80	–	–	5.19	6	15.58
Specimen	1.42	2.15	3.3	9.4	180	130	30	–	–

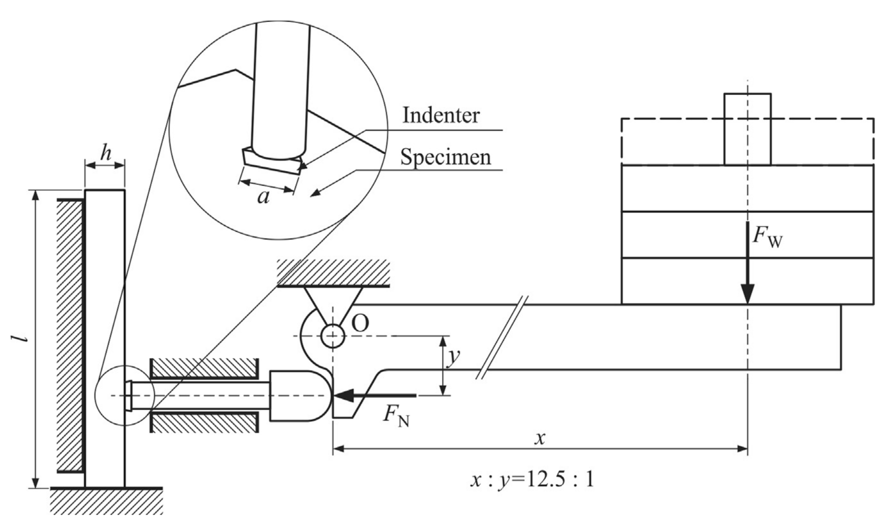

Fig. 3. Experimental indenting process.

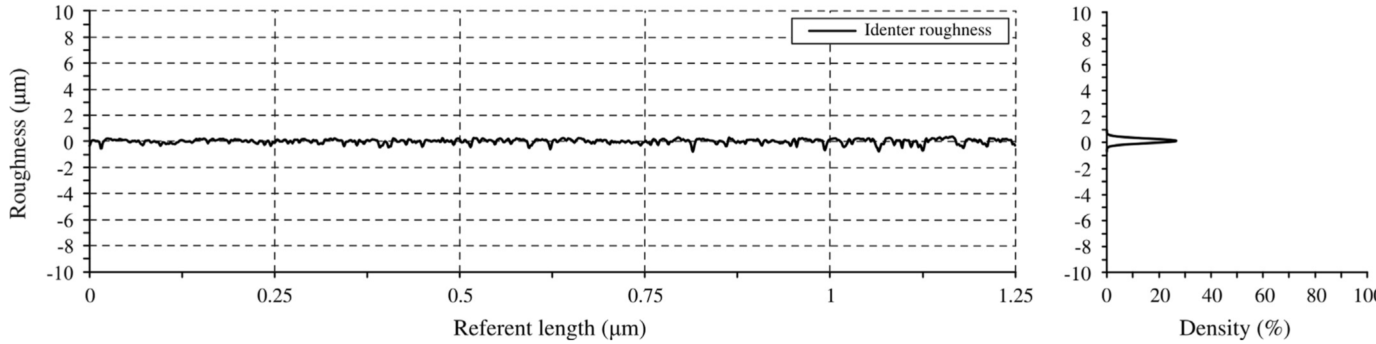

Fig. 4. Effective profile of indenter and Gaussian distributions of heights.

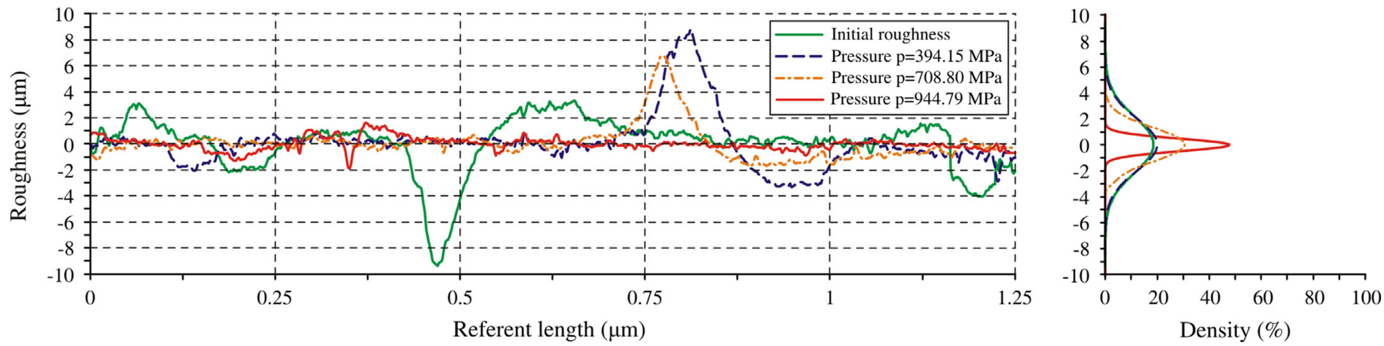

Fig. 5. Superimposed roughness profile of specimen and the Gaussian distribution of heights.

Table 3
The measured values of the roughness parameters for various nominal pressures.

	$p=0$ (MPa)	$p=394.1$ (MPa)	$p=708.8$ (MPa)	$p=944.8$ (MPa)
R_a (μm)	1.42	1.11	0.73	0.34
R_q (μm)	2.15	2.08	1.30	0.48
R_p (μm)	3.33	8.81	6.74	1.60
R_v (μm)	9.40	3.30	1.72	1.88

and it was shown that the level of significance for all surface roughness data was $p<0.01$. Therefore, the null hypothesis, i.e., that the heights of the surface roughness peaks follow a Gaussian distribution, was not rejected. Measurements of the surface roughness profiles of the treated surface were superimposed (Fig. 5), which allowed a visual assessment of the changes in the surface roughness peak heights due to the contact pressure. A comparison of the diagrams shown in Fig. 4 and Fig. 5 leads to the conclusion that greater nominal pressures cause the surface roughness of the specimen to converge to that of the indenter. It was also confirmed (from Fig. 5) that some asperities persist, even under very high loads, which is consistent with the results reported by Pullen and Williamson [41] and a number of other researchers [40,42,43]. Table 3 shows the values of the chosen R-parameters as a function of the nominal pressure.

Fig. 6 shows the dispersion of the R_a measurements for each of the nominal pressure values. The diagram provides insight into the dependence between the absolute deviation of R_a and the nominal pressure, p. A regression analysis of the experimental data showed a strong correlation, where the R_a decreases with increasing nominal pressure. The value of the correlation coefficient, R_c, was -0.95.

The previous data analysis showed reduced R_a values in all of the samples. However, to define the boundaries where the transition from elastic to plastic deformation occurs, it is necessary to consider the relationship between the surface roughness and the critical value of the nominal pressure. The basic relationship

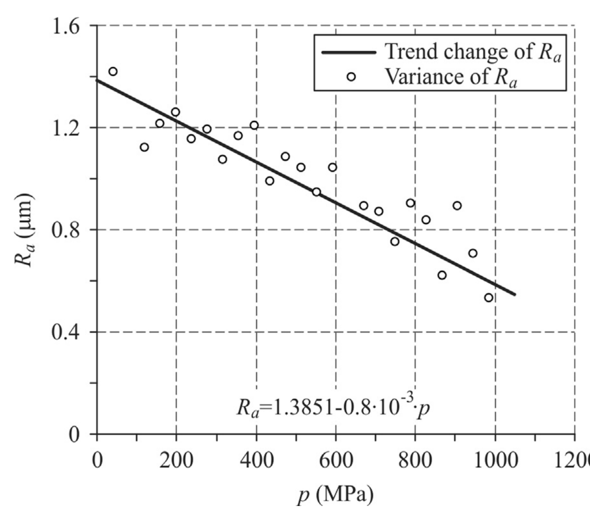

Fig. 6. Variance of R_a and the linear regression function.

between the material properties and the contact surface topography is the plasticity index, ψ. According to Greenwood and Williamson [25], the plasticity index entirely defines the deformation behaviour, meaning that the load itself is of little consequence. Theoretically, the plasticity index, ψ, can take any value between 0 and ∞. When $\psi=0.6$–1.0, any type of deformation is possible. For $\psi<0.6$, most of the peaks will undergo elastic deformation, and plastic deformation occurs only if the contact surfaces are subjected to extremely high nominal pressures. When $\psi>1.0$, plastic material flow takes place, even at very low nominal pressures. Many surfaces have a plasticity index greater than 1.0, which means that, except for very smooth surfaces, roughness peaks will undergo plastic flow, even under small loads.

Table 4 shows the change in the plasticity index ψ, calculated from Eq. (4), due to the application of various nominal loads. Considering the high E^*/H ratio, which indicates a high plasticity

Table 4
Variation of the plasticity index ψ with nominal pressure p.

p (MPa)	40.1	158.1	236.8	394.1	512.1	669.4	708.8	787.4	944.8
σ (μm)	2.10	2.03	1.84	2.08	1.74	1.52	1.30	1.45	0.48
r (μm)	11.9	11.1	11.6	16.4	20.6	17.2	26.1	28.2	34.2
ψ (−)	27.6	28.1	26.2	23.4	19.1	19.5	14.7	14.9	7.8

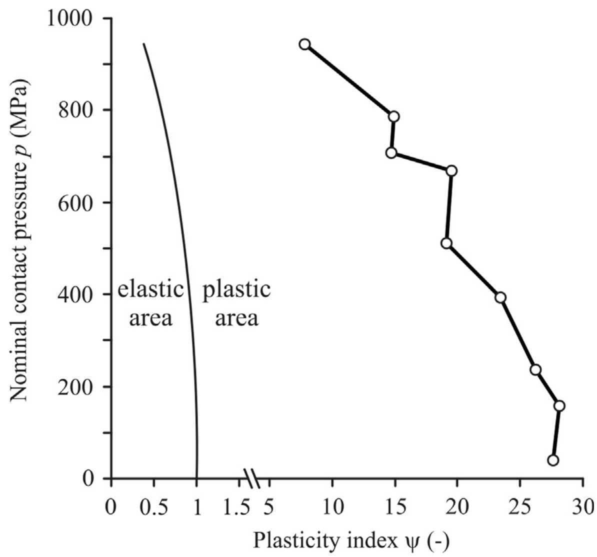

Fig. 7. Changes in the plasticity index due to variation of the nominal pressure based on the onset of plastic deformation of surface roughness peaks.

index, it can be concluded that the peaks deform regardless of the nominal pressure applied (Fig. 7).

The successful treatment of a surface by plastic forming requires, among other things, information on the properties of the materials being treated. The most important property is the yield stress, which determines when the transition from elastic to plastic deformation occurs. In materials that undergo cold treatment (deformation strengthening), the yield stress varies and directly depends on the degree of deformation. This is why a unique relationship between these two factors must be established over a wide range. For this purpose, stress–strain curves must be used, which graphically show the functional relationship between the yield stress and the particular deformation.

5. Conclusions

The results obtained in this investigation demonstrate that plastic deformation contributes to the reduction of surface roughness. An analysis of the surface roughness data and a comparison of the surface profiles obtained for plastic deformation under various loads (Fig. 5) led to the conclusion that the surface roughness of the specimen converges to that of the indenter. This method of surface treatment can be of particular importance when treating surfaces of rough initial quality, where the values of R_a are high.

Fig. 5 reveals the reduction of the surface roughness peak heights relative to the initial roughness as the nominal pressure rises. However, bearing in mind the importance of the plasticity index, ψ, for most surfaces the magnitude of the load is of little consequence to the development of deformations. The plasticity index is directly related to the ratio between the modulus of elasticity and the actual material hardness, i.e., E^*/H. If the ratio E^*/H is low, the plasticity index will be low and the resulting deformation will be elastic. Conversely, if the ratio E^*/H is high, the plasticity index will be high, leading to plastic deformation. Considering the values of ψ obtained in this study (Table 4), which were within the range of 7.8–28.1, it is clear that the roughness peaks plastically deform regardless of the actual loads. The surface roughness of the specimen after the completion of the plastic forming process will depend, among other factors, on the surface roughness of the indenter. The experiments were conducted with an indenter of low roughness (R_a=0.13 μm) and the roughness of the specimen was reduced from R_a=1.42 μm to R_a=0.34 μm, so the prediction that the indentation process would produce a smoother workpiece was correct. It should be noted that even under the highest load, the roughness of the specimen was 2.42 times that of the indenter. The reason is that even under very heavy loads, the asperities on rough surfaces are not completely flattened because of asperity persistence [40–43].

Future investigations regarding the reduction of surface roughness by plastic deformation using a flat surface as the indenter could go in numerous directions. Experiments could be performed on specimens of various materials and surface roughnesses. Furthermore, it would be interesting to investigate the homogeneity of the micro-scale structure of the surface to establish whether it exclusively results from the deformation of peaks or if the breaking of peaks under load is also of consequence. The influence of temperature on the deformation process is also a potential direction for future investigation.

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